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ABSTRACT

Five sets of activities for students are included in this document. Each is designed for use in junior high and secondary school mathematics instruction. The first two are concerned with the properties of polygons. The third involves statistical decision making using basketball statistics as examples. The fourth is related to the perimeter and area of squares and rectangles. The fifth describes the uses of directed graphs. Each set contains a variety of activities, an extension section, and an answer key. (CW)

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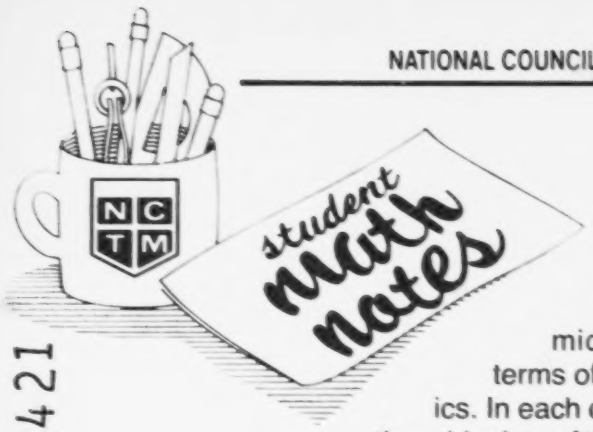

Midpoint Madness

The concept of midpoint is very rich in terms of its usefulness in mathematics. In each of the following quadrilaterals find the midpoints of the sides and label them consecutively *N*, *C*, *T*, and *M*. Then draw the segments forming the polygon *NCTM*.

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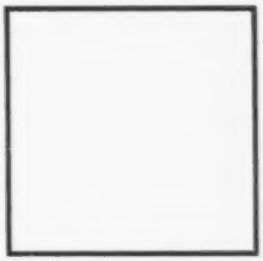
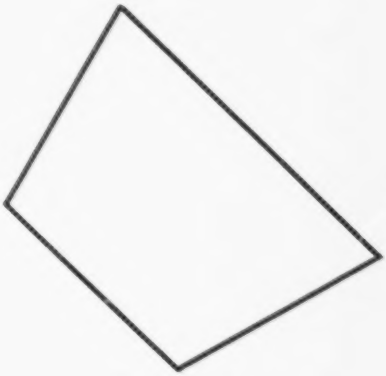


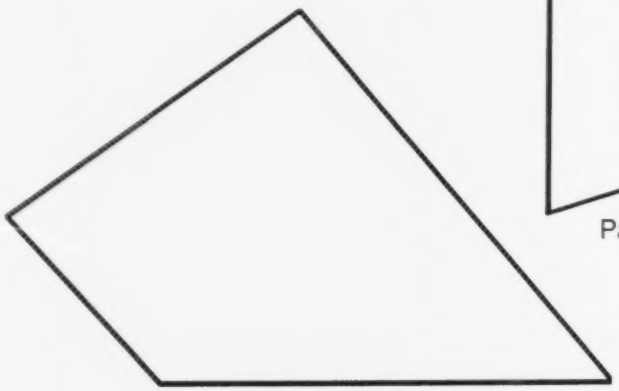
Square

Rectangle

Parallelogram



Trapezoid



General quadrilateral

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What appears to be true about all the *NCTM* polygons? _____

_____

What also appears to be true about the relationship between the area of each *NCTM* polygon and the area of its original quadrilateral? _____

_____

SE 050 259

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For centuries mathematics students have studied a theorem stating that connecting the midpoints of consecutive sides of any quadrilateral will always form a parallelogram and that the area of the parallelogram is one-half the area of the original quadrilateral.

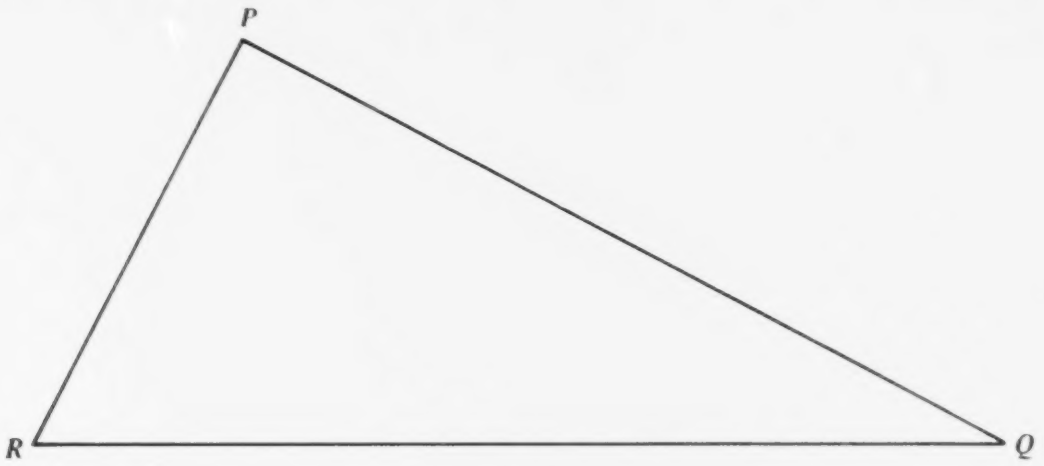


Fig. 1

In $\triangle PQR$, locate the midpoints of sides $\overline{PQ}$, $\overline{QR}$, and $\overline{RP}$ and label them A_1, B_1, and C_1, respectively. Connect these midpoints, forming $\triangle A_1B_1C_1$.

- How do $m\angle P$ and $m\angle A_1B_1C_1$ compare? _____
- How do $m\angle Q$ and $m\angle A_1C_1B_1$ compare? _____
- How do $\triangle PQR$ and $\triangle B_1C_1A_1$ compare in terms of shape? _____
- How do they compare in terms of area? _____
- What kinds of quadrilaterals are polygons $A_1B_1C_1P$, $A_1QB_1C_1$, and $A_1B_1RC_1$?

  _____

  Why? _____
- What kinds of quadrilaterals are polygons A_1B_1RP, C_1A_1QR, and B_1C_1PQ?

  _____

  Why? _____

Find the midpoints of the sides of $\triangle A_1B_1C_1$ and name them A_2, B_2, and C_2. (You may label them in any order.)

- How do $\triangle PQR$, $\triangle A_1B_1C_1$, and $\triangle A_2B_2C_2$ compare in terms of shapes?

  _____

  How do they compare in terms of areas? _____

Repeat the process of successively finding midpoints of the sides of the newly formed interior triangles, labeling them A_i, B_i, and C_i as the number of triangles increases from 1 to 2 to 3 to i.

- What do you notice about the points A_i, B_i, and C_i as i increases?

  _____

Draw in the medians $\overline{RA_1}$, $\overline{PB_1}$, and $\overline{QC_1}$. Label their point of intersection O.

- What do you notice about O and the points A_i, B_i, and C_i as i increases?

  _____

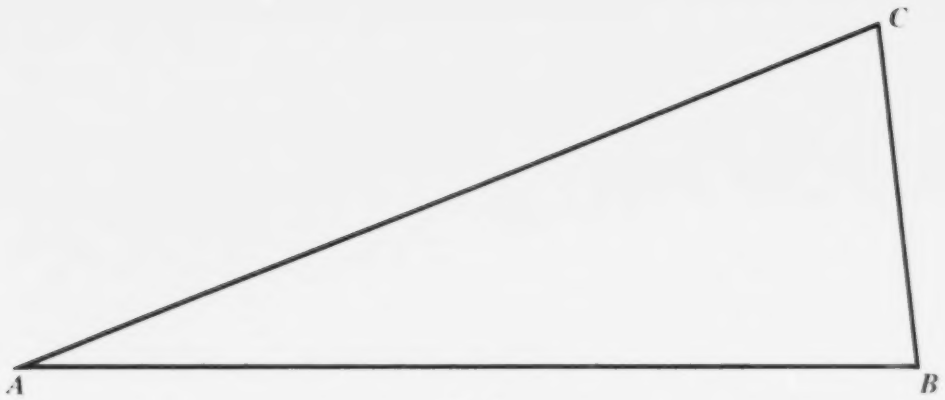


Fig. 2

In $\triangle ABC$, locate D_1, the midpoint of $\overline{AC}$, and E_1, the midpoint of $\overline{BC}$. Draw $\overline{D_1E_1}$; measure $\overline{D_1E_1}$ and $\overline{AB}$.

$D_1E_1 =$ _____; $AB =$ _____.

- How does D_1E_1 compare to AB? _____

  Draw $\overline{D_1B}$ and $\overline{E_1A}$; label the point of intersection F_1.

Locate D_2, the midpoint of $\overline{AF_1}$, and E_2, the midpoint of $\overline{BF_1}$. Draw $\overline{D_2E_2}$; measure $\overline{D_2E_2}$.

$D_2E_2 =$ _____.

- How does D_2E_2 compare to D_1E_1? _____
- How does D_2E_2 compare to AB? _____

Draw $\overline{D_2B}$ and $\overline{E_2A}$; label the point of intersection F_2.

Locate D_3, the midpoint of $\overline{AF_2}$, and E_3, the midpoint of $\overline{BF_2}$. Draw $\overline{D_3E_3}$; measure $\overline{D_3E_3}$. $D_3E_3 =$ _____.

- How does D_3E_3 compare to D_1E_1 and D_2E_2?

  _____

- How does D_3E_3 compare to AB?

  _____

- If we continue, how would the measures of all the segments D_iE_i compare to one another?

  _____

- If we continue, how would the measures of all the segments D_iE_i compare to the measure of $\overline{AB}$?

  _____

- What do you notice about the points F_i as i increases?

  _____

Draw $\overline{D_1D_2}$, $\overline{D_2D_3}$, $\overline{E_1E_2}$, $\overline{E_2E_3}$, and so on.

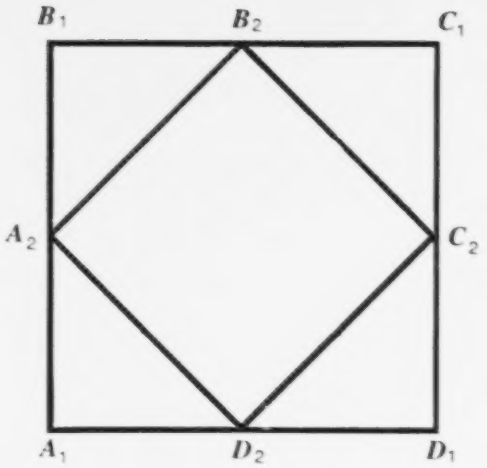
- What kind of quadrilateral is $D_1D_2E_2E_1$ or $D_2D_3E_3E_2$ or $D_1D_3E_3E_1$? _____

  Why? _____

- What kind of quadrilateral is AD_1E_1B or AD_2E_2B or AD_3E_3B? _____

  Why? _____

The midpoints of each side were located in the unit square $A_1B_1C_1D_1$. The square $A_2B_2C_2D_2$ was then constructed.



How does the area of $A_2B_2C_2D_2$ compare to the area of $A_1B_1C_1D_1$?

Now locate the midpoints of each side of $A_2B_2C_2D_2$ and label them A_3, B_3, C_3, and D_3 to form the square $A_3B_3C_3D_3$.

How does the area of $A_3B_3C_3D_3$ compare to the area of $A_1B_1C_1D_1$?

Continue to generate several more squares using the process described previously. How does the area of $A_5B_5C_5D_5$ compare to the area of $A_1B_1C_1D_1$?

How does the area of $A_6B_6C_6D_6$ compare to the area of $A_1B_1C_1D_1$? _____

What is the area of $A_iB_iC_iD_i$ approaching as i gets larger? _____

With a colored pencil, connect segments A_1A_2, A_2A_3, A_3A_4, and A_4A_5.

1. What is the sum of the measures of these four segments? _____
2. What is the sum of the measures of n segments (as n gets extremely large)? _____
3. Assume that the squares are generated ad infinitum. What is the measure of $A_1A_2 + A_2A_3 + A_3A_4 + \ldots$?

Did you know that . . .

- the point O in figure 1 is called the centroid of triangle PQR? In physics, centroids are known as the center of mass.
- the sequence ½, ¼, ⅛, . . . , ½^n is known as a geometric sequence?
- when the terms of a geometric sequence are added together, a geometric series is formed?
- the sum of the geometric series 1 + ½ + ¼ + ⅛ + . . . is 2?

Can you . . .

- prove that the $NCTM$ polygons are all parallelograms?
- prove that the area of each $NCTM$ polygon is half the area of its original quadrilateral?
- prove that the area of $\triangle A_1B_1C_1$ in figure 1 is one-fourth the area of $\triangle PQR$?
- prove that the area of $\triangle CD_1E_1$ in figure 2 is one-fourth the area of $\triangle ABC$?

Solutions

1. $2(1 - (\frac{1}{2})^n)$; 2. $\frac{10 + 5\sqrt{2}}{16}$; 3. $\frac{2 + \sqrt{2}}{3}$

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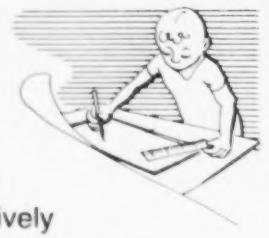
student math notes

Midpoint Madness



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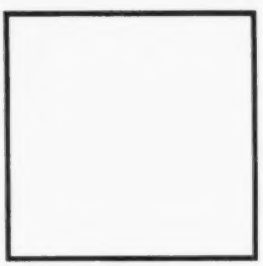
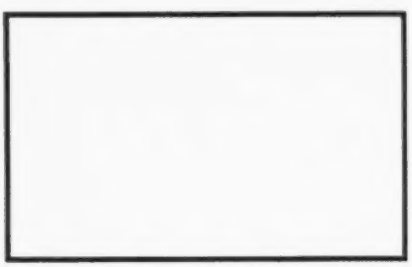
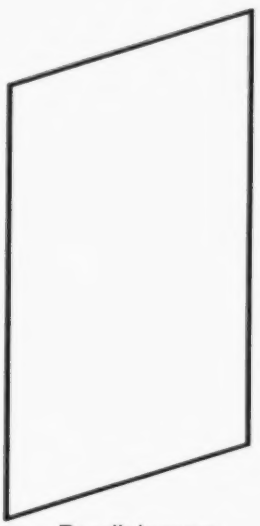


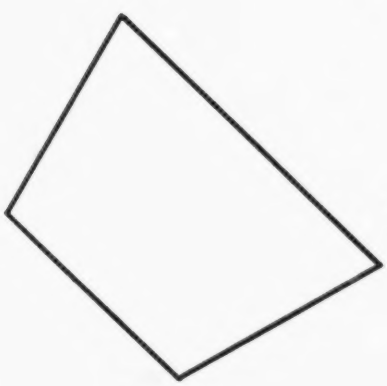
Square



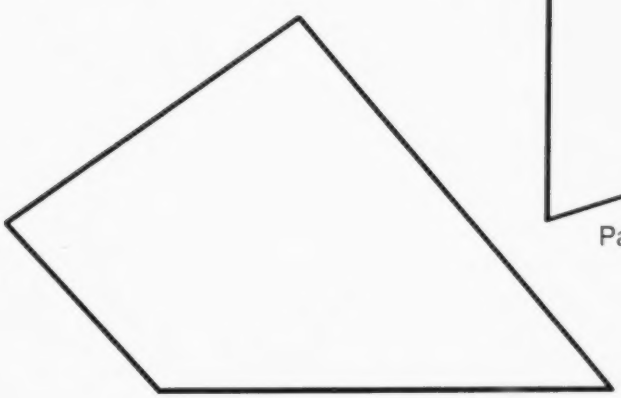
Rectangle



Parallelogram



Trapezoid



General quadrilateral

What appears to be true about all the ***NCTM*** polygons? _____

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What also appears to be true about the relationship between the area of each ***NCTM*** polygon and the area of its original quadrilateral? _____

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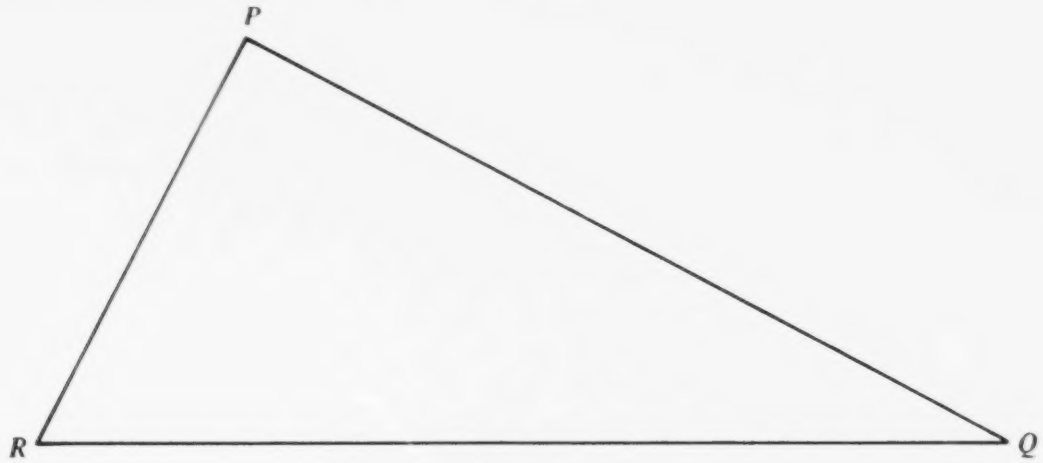

Fig. 1

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- How do $m\angle P$ and $m\angle A_1B_1C_1$ compare? _____
- How do $m\angle Q$ and $m\angle A_1C_1B_1$ compare? _____
- How do $\triangle PQR$ and $\triangle B_1C_1A_1$ compare in terms of shape? _____
- How do they compare in terms of area? _____
- What kinds of quadrilaterals are polygons $A_1B_1C_1P$, $A_1QB_1C_1$, and $A_1B_1RC_1$?

  _____

  Why? _____
- What kinds of quadrilaterals are polygons A_1B_1RP, C_1A_1QR, and B_1C_1PQ?

  _____

  Why? _____

Find the midpoints of the sides of $\triangle A_1B_1C_1$ and name them A_2, B_2, and C_2. (You may label them in any order.)

- How do $\triangle PQR$, $\triangle A_1B_1C_1$, and $\triangle A_2B_2C_2$ compare in terms of shapes?

  _____

  How do they compare in terms of areas? _____

Repeat the process of successively finding midpoints of the sides of the newly formed interior triangles, labeling them A_i, B_i, and C_i as the number of triangles increases from 1 to 2 to 3 to i.

- What do you notice about the points A_i, B_i, and C_i as i increases?

  _____

Draw in the medians $\overline{RA_1}$, $\overline{PB_1}$, and $\overline{QC_1}$. Label their point of intersection O.

- What do you notice about O and the points A_i, B_i, and C_i as i increases?

  _____

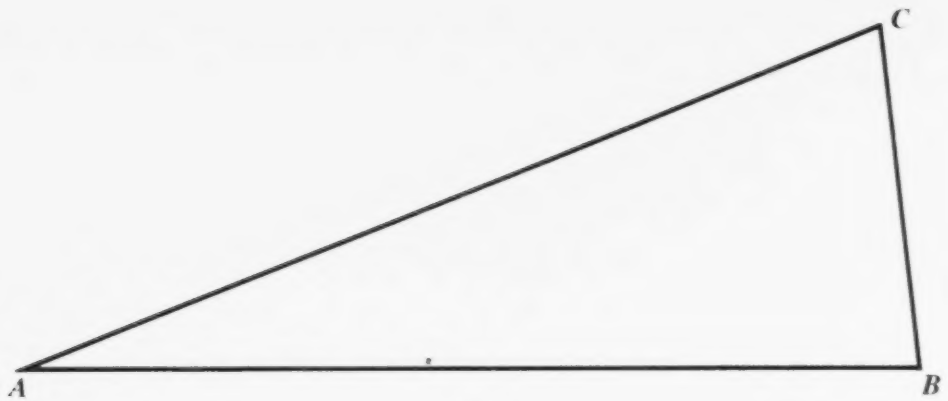


Fig. 2

In $\triangle ABC$, locate D_1, the midpoint of $\overline{AC}$, and E_1, the midpoint of $\overline{BC}$. Draw $\overline{D_1E_1}$; measure $\overline{D_1E_1}$ and $\overline{AB}$.

$D_1E_1 =$ _____; $AB =$ _____.

- How does D_1E_1 compare to AB? _____

Draw $\overline{D_1B}$ and $\overline{E_1A}$; label the point of intersection F_1.

Locate D_2, the midpoint of $\overline{AF_1}$, and E_2, the midpoint of $\overline{BF_1}$. Draw $\overline{D_2E_2}$; measure $\overline{D_2E_2}$.

$D_2E_2 =$ _____.

- How does D_2E_2 compare to D_1E_1? _____
- How does D_2E_2 compare to AB? _____

Draw $\overline{D_2B}$ and $\overline{E_2A}$; label the point of intersection F_2.

Locate D_3, the midpoint of $\overline{AF_2}$, and E_3, the midpoint of $\overline{BF_2}$. Draw $\overline{D_3E_3}$; measure $\overline{D_3E_3}$. $D_3E_3 =$ _____.

- How does D_3E_3 compare to D_1E_1 and D_2E_2?

_____

- How does D_3E_3 compare to AB?

_____

- If we continue, how would the measures of all the segments D_iE_i compare to one another?

_____

- If we continue, how would the measures of all the segments D_iE_i compare to the measure of $\overline{AB}$?

_____

- What do you notice about the points F_i as i increases?

_____

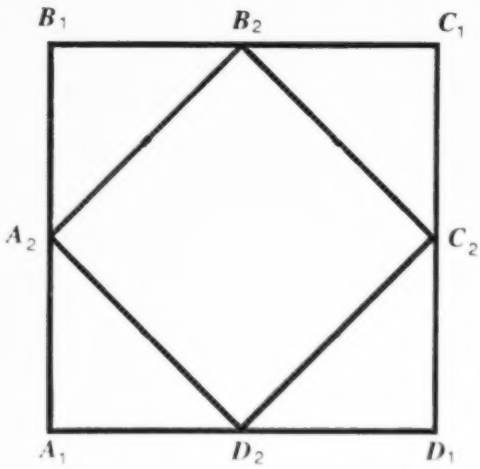
Draw $\overline{D_1D_2}$, $\overline{D_2D_3}$, $\overline{E_1E_2}$, $\overline{E_2E_3}$, and so on.

- What kind of quadrilateral is $D_1D_2E_2E_1$ or $D_2D_3E_3E_2$ or $D_1D_3E_3E_1$? _____

  Why? _____
- What kind of quadrilateral is AD_1E_1B or AD_2E_2B or AD_3E_3B? _____

  Why? _____

The midpoints of each side were located in the unit square $A_1B_1C_1D_1$. The square $A_2B_2C_2D_2$ was then constructed.



How does the area of $A_2B_2C_2D_2$ compare to the area of $A_1B_1C_1D_1$?

Now locate the midpoints of each side of $A_2B_2C_2D_2$ and label them A_3, B_3, C_3, and D_3 to form the square $A_3B_3C_3D_3$.

How does the area of $A_3B_3C_3D_3$ compare to the area of $A_1B_1C_1D_1$?

Continue to generate several more squares using the process described previously. How does the area of $A_5B_5C_5D_5$ compare to the area of $A_1B_1C_1D_1$?

How does the area of $A_9B_9C_9D_9$ compare to the area of $A_1B_1C_1D_1$? _____

What is the area of $A_iB_iC_iD_i$ approaching as i gets larger? _____

With a colored pencil, connect segments A_1A_2, A_2A_3, A_3A_4, and A_4A_5.

1. What is the sum of the measures of these four segments? _____
2. What is the sum of the measures of n segments (as n gets extremely large)? _____
3. Assume that the squares are generated ad infinitum. What is the measure of $A_1A_2 + A_2A_3 + A_3A_4 + \ldots$?

Did you know that . . .

- the point O in figure 1 is called the centroid of triangle PQR? In physics, centroids are known as the center of mass.
- the sequence ½, ¼, ⅛, . . . , $\frac{1}{2^n}$ is known as a geometric sequence?
- when the terms of a geometric sequence are added together, a geometric series is formed?
- the sum of the geometric series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$ is 2?

Can you . . .

- prove that the $NCTM$ polygons are all parallelograms?
- prove that the area of each $NCTM$ polygon is half the area of its original quadrilateral?
- prove that the area of $\triangle A_1B_1C_1$ in figure 1 is one-fourth the area of $\triangle PQR$?
- prove that the area of $\triangle CD_1E_1$ in figure 2 is one-fourth the area of $\triangle ABC$?

Solutions

1. $2(1 - (\frac{1}{2})^n)$; 2. $\frac{10 + 5\sqrt{2}}{16}$; 3. $\frac{2 + \sqrt{2}}{3}$

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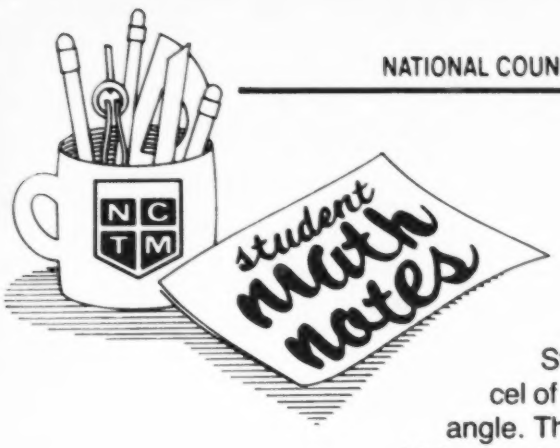
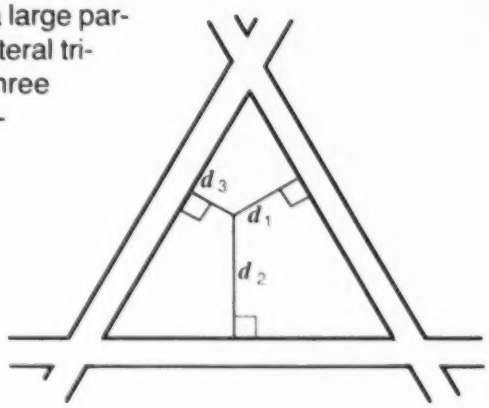


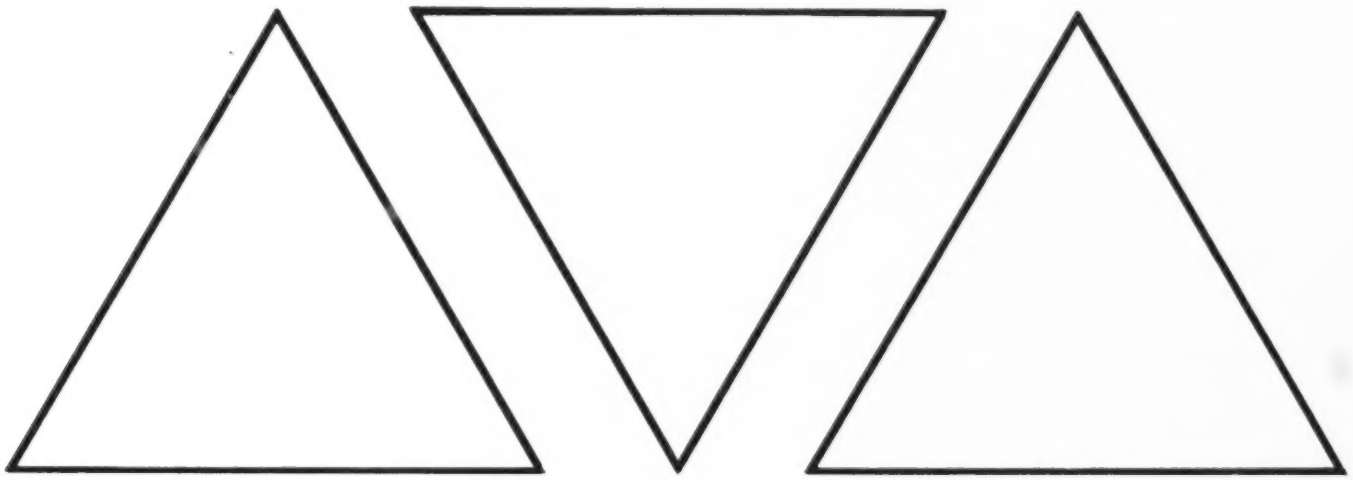
Minimum Distance

Stu Dent recently inherited a large parcel of land shaped like an equilateral triangle. The land's boundaries are three highways, intersecting to form the triangle's three vertices. However, for Stu to receive the land, the will required that he build a house on the triangular parcel of land and also that he build three roads from the house, each perpendicular to one of the three highways. Furthermore, the will required that the sum of the distances, $d_1 + d_2 + d_3$, from the house to the highways *must* be the **smallest** possible.
Where should Stu put his house?



Let's Explore Some Possibilities for Stu

Using a ruler and protractor, locate possible points—housing sites—on or within the triangles below and construct the three roads from the house perpendicular to the highways. Measure and record the **sum** of the three distances under each triangle.

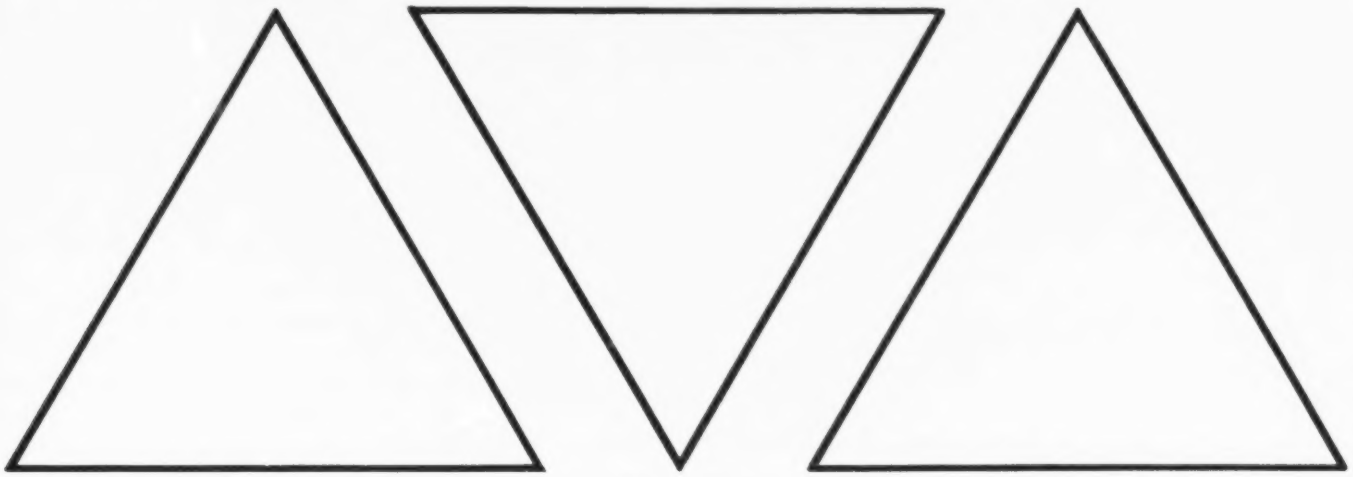


Sum = ___ Sum = ___ Sum = ___

The editors wish to thank Jerry L. Johnson, Western Washington University, Bellingham, WA 98225, for writing this issue of the *NCTM Student Math Notes*.

Continuing Your Explorations

Now try some additional points. Be sure to try special points, such as the intersection of the medians, a vertex, or a point on the edge of the parcel of land.



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- What is the sum of the distances whenever the point chosen is—

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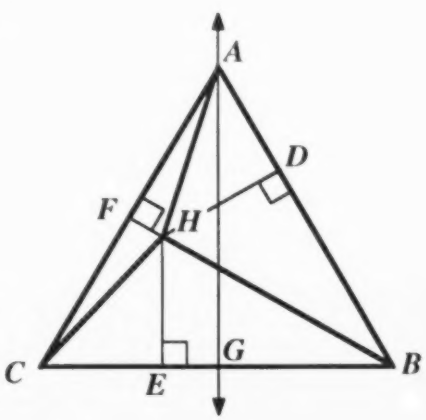

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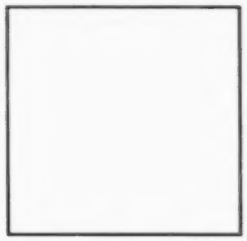

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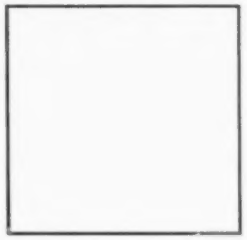

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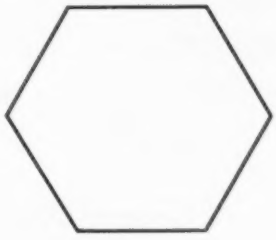
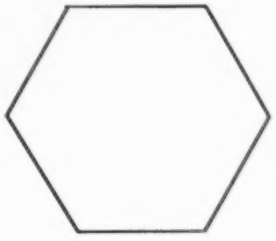


Sum = ___



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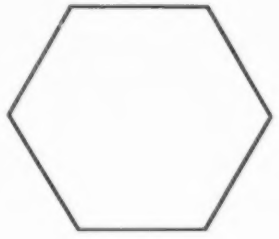

Sum = ___

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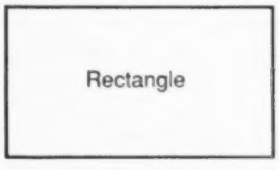

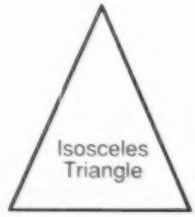

- What conclusions can you make regarding the best location for Stu's house in these new shapes? _____

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Can you . . .

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- write a Logo program that will simulate the original problem visually, calculating the respective lengths and sum as well?
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$$HA + HB + HC \geq 2(HD + HE + HF)?$$

The equality holds if and only if $\triangle ABC$ is equilateral and the point H is its circumcenter. This conjecture was first proved in 1937 by L. J. Mordell and D. R. Barrow.

Solutions

Page 3: top (*a*) Any point in the square
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bottom Any point will work for the rectangle or any regular polygon. No obvious (or easy) answer exists for other shapes.

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Thank you, attentive readers!

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Editorial Coordinator: **Joan Armistead**
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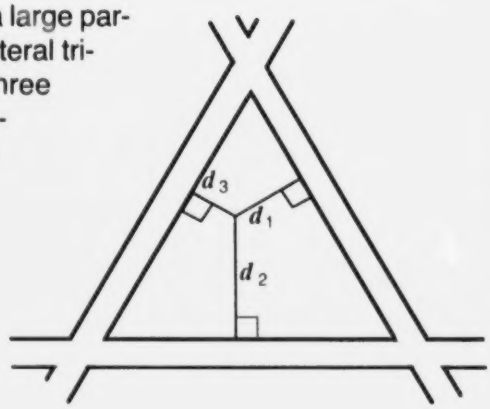
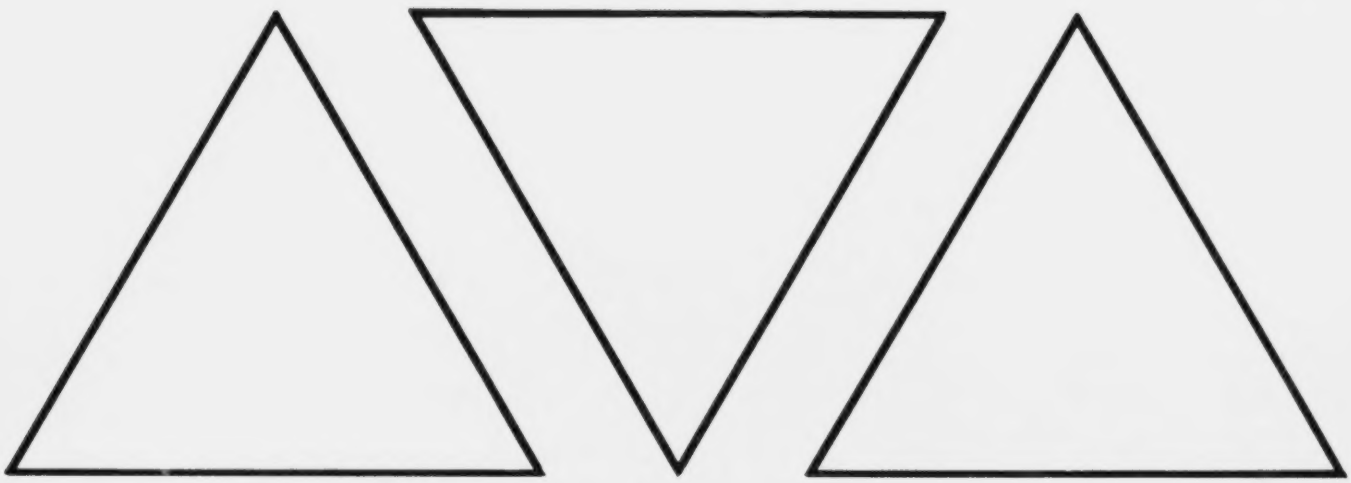
Minimum Distance

Stu Dent recently inherited a large parcel of land shaped like an equilateral triangle. The land's boundaries are three highways, intersecting to form the triangle's three vertices. However, for Stu to receive the land, the will required that he build a house on the triangular parcel of land and also that he build three roads from the house, each perpendicular to one of the three highways. Furthermore, the will required that the sum of the distances, $d_1 + d_2 + d_3$, from the house to the highways *must* be the **smallest** possible. Where should Stu put his house?



Let's Explore Some Possibilities for Stu

Using a ruler and protractor, locate possible points—housing sites—on or within the triangles below and construct the three roads from the house perpendicular to the highways. Measure and record the **sum** of the three distances under each triangle.



Sum = ___ Sum = ___ Sum = ___

The editors wish to thank Jerry L. Johnson, Western Washington University, Bellingham, WA 98225, for writing this issue of the *NCTM Student Math Notes*.

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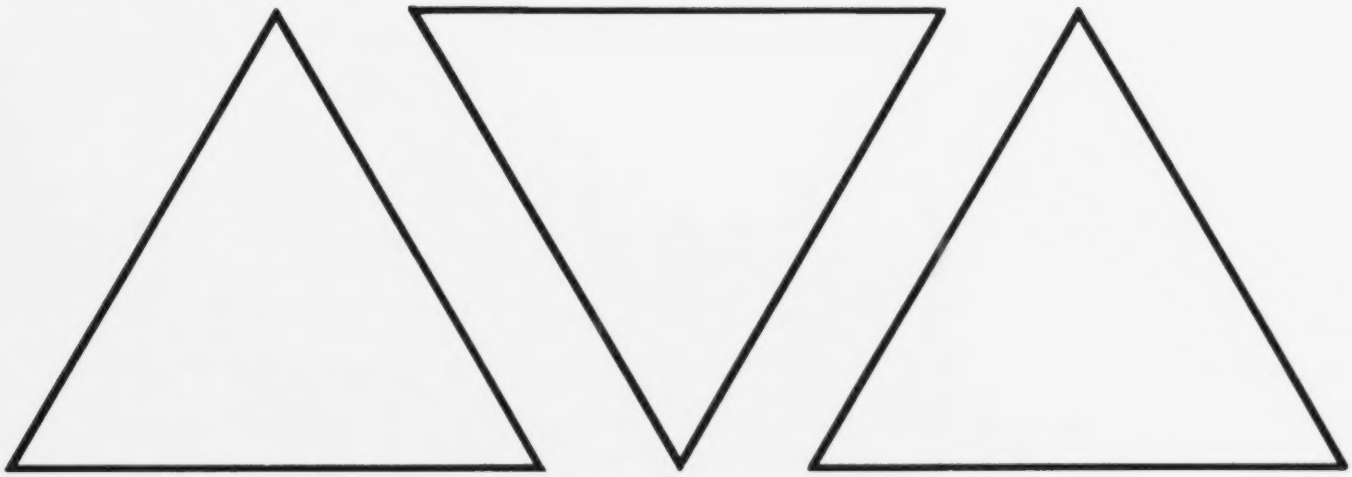
Now try some additional points. Be sure to try special points, such as the intersection of the medians, a vertex, or a point on the edge of the parcel of land.



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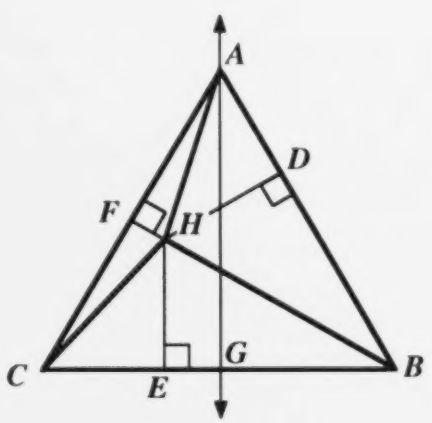

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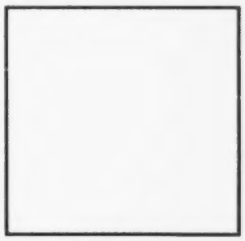

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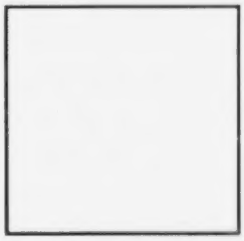

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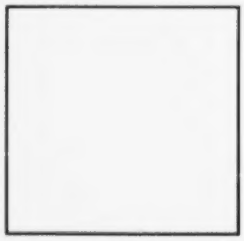

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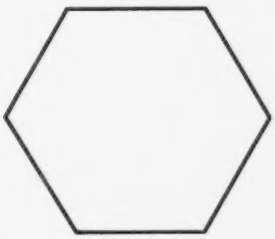

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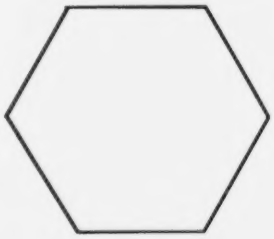

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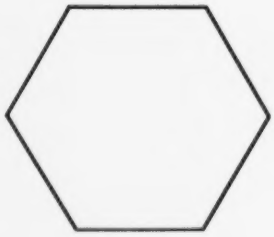

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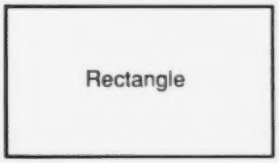

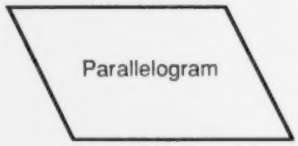

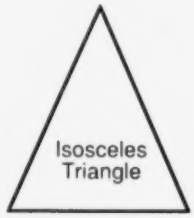

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On the Road with NCTM: Future Meeting Sites Are Announced

Don't pack your suitcase yet, but consider the possibilities: NCTM has announced its schedule of annual meetings and regional conferences for the next few years.

Aside from Chicago, the location of next month's 66th Annual Meeting, other locations for annual meetings in upcoming years are as follows:

- 67th: **Orlando,** 12–14 April 1989
- 68th: **Salt Lake City,** 18–21 April 1990
- 69th: **New Orleans,** 17–20 April 1991
- 70th: **Nashville,** 1–4 April 1992
- 71st: **Seattle,** 31 March–3 April 1993
- 72d: **Indianapolis,** 13–16 April 1994

Nashville, the home of country music, will be the site of the 70th Annual Meeting in 1992.

Here's the schedule of regional conferences for 1988–89:

- **Pittsburgh,** 12–14 October
- **St. Louis,** 27–29 October
- **Baton Rouge,** 3–5 November
- **Boston,** 1–3 December
- **San Jose,** 23–25 February (1989)
- **Helena,** 2–4 March (1989)
- **Grand Rapids,** 9–11 March (1989)
- **Omaha,** 16–18 March (1989)

For those who keep multi-year appointment books, here are the locations of regional conferences in 1989–90:

- **Rapid City, South Dakota,** 5–7 October 1989
- **San Juan,** 12–14 October 1989
- **Denver,** 18–22 October 1989
- **Saskatoon, Saskatchewan,** 26–28 October 1989
- **Philadelphia,** 30 November–2 December 1989
- **Chattanooga,** 14–17 March 1990
- **Hamilton, Ontario,** 9–12 May 1990

Finally, for those who really plan ahead, the following is a list of meetings to be held in 1990–91:

- **Honolulu,** Summer 1990
- **East Rutherford/Secaucus,** New Jersey, Fall 1990
- **Memphis,** Fall 1990
- **South Bend, Indiana,** Fall 1990
- **Wichita, Kansas,** Fall 1990
- **Madison, Wisconsin,** Fall/Winter 1990–91
- **Sacramento, California,** Spring 1991

Help Plan an NCTM Regional Conference

NCTM's Conventions and Conferences Committee invites you to participate in the plans being made for upcoming regional conferences. If you know a good speaker who can contribute to one of the meetings, or if you have any suggestions about types of sessions, scheduling, or other aspects of the meetings, please send in your ideas. Just ask for a recommendation form from the Headquarters Office—the form contains the names and addresses of program chairmen so you can easily send your ideas to the right person. (You should, however, send in your suggestions no later than 13 months before the meeting.)

For the locations of upcoming regional conferences, see the related article at the top of this page. Plan now to get involved in one or more of these important events.

This six-spired temple dominates Temple Square in Salt Lake City. (Courtesy of Salt Lake Convention and Visitors Bureau)

NCTM Arranges Calculator Sessions at NSTA and NAESP Conventions

NCTM members are scheduled to make presentations on the use of calculators in the mathematics classroom at the 1988 annual meetings of the National Science Teachers Association (NSTA) and the National Association of Elementary School Principals (NAESP).

On 7–10 April in St. Louis, Richard Lodholz of the Parkway School District in Creve Coeur, Missouri, will discuss "Improving Science Education in the Mathematics Class." He will focus on developmental lessons requiring an investigative approach that emphasizes process development and that is made possible by the technology of the handheld calculator. The presentation reinforces NCTM's position statement urging the use of calculators in the classroom.

Also in line with NCTM's advocacy of calculators for all students will be the "brainstorming session" led by Kay Gilliland at the 1988 convention of the NAESP, to be held in San Francisco, California, 16–20 April. Gilliland, who is director of EQUALS in Computer Technology at the University of California at Berkeley, will explain NCTM's endorsement of calculators and outline the progress that various states are making in incorporating their use into the curriculum.

Go Around the World in 80 Ways with NCTM Travel Programs

It's time for your pop quiz in geography. Ready? Which of the following continents do NCTM tours and charter flights *not* reach?

a) Asia *b*) Europe *c*) Africa
d) Antarctica *e*) Australia

If you answered "*d*," then you're correct. NCTM does not arrange travel to Antarctica—yet. But you can see Asia, Africa, Europe, or Australia on one of the tours or charter flights arranged by the NCTM travel office.

What's your fancy? Rome? Mysterious Egypt? Do you like to follow your own whims when sightseeing? Or would you rather enjoy the company of a group in a guided tour? Either way, NCTM has something for you.

Charter Flights

If you prefer to set your own itinerary, then take advantage of the charter and discount flights that NCTM arranges through the National Center for Education Travel. You can fly from 62 cities in the U.S. to a host of cities around the globe. For information about round-trip airfares from points in the U.S. to such cities as Paris, Cairo, Tokyo, and Rio de Janeiro—and many others—fill out and mail the coupon below to George Marucci at the NCET.

Complete Tours

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- Hong Kong and Beijing (2 weeks), $2099
- Around the World (26 days), $2999

Please send me your brochure with complete details.

Name _____

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Mail to Ed Nevins, NCTM Travel Coordinator, P.O. Box 366, Strasburg, VA 22657 (703/465-4211). Prices subject to change, but will be confirmed once bookings are made.
NCTM, NB 3/88

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- From 62 cities in the U.S. to London, Glasgow, Shannon, Paris, Nice, Brussels, Amsterdam, Frankfurt, Munich, Hamburg, Düsseldorf, Vienna, Copenhagen, Geneva, Zurich, Rome, Milan, Athens, Tel Aviv, Cairo, Madrid, Malaga, Lisbon, Rio de Janeiro, Tokyo, Hong Kong, Taipei, Bangkok, Singapore, Beijing, Shanghai, Auckland, Sydney, Karachi, New Delhi, Bombay, eastern Europe.

- For NCTM members, their families, and friends

Name _____

Address _____

- For free brochure, send this coupon to NCET, 15109 Redgate Dr., Silver Spring, MD 20904 (301/384-5000).
NCTM, NB 3/88

student math notes

Statistical Decision Making

A panel of five experts rated the centers in the National Basketball Association (NBA) on five characteristics: aggressiveness, shooting range, teamwork, offense, and defense. The players were rated on each characteristic from 10 for excellent to 1 for poor. Table 1 gives the totals in each category for ten of the players.

Table 1

Player	Aggressiveness	Ranking	Shooting Range	Ranking	Teamwork	Ranking	Offense	Ranking	Defense	Ranking	Total of Rankings	Order
Brad Daugherty (Cleveland Cavaliers)	41		37		34	5	38		41			
Patrick Ewing (New York Knicks)	47		34		36	2	45		39			
Artis Gilmore (Chicago Bulls)	41		34		32	8	32		33			
Kareem Abdul Jabbar (Los Angeles Lakers)	33		38		48	1	48		35			
Bill Laimbeer (Detroit Pistons)	45		42		35	3	36		31			
Moses Malone (Washington Bullets)	49		37		27	10	46		36			
Akeem Olajuwon (Houston Rockets)	48		36		33	7	48		50			
Robert Parrish (Boston Celtics)	36		28		35	3	39		40			
Tree Rollins (Atlanta Hawks)	46		26		31	9	33		38			
Jack Sikma (Milwaukee Bucks)	33		43		34	5	36		28			

Using these results, who would you choose as the number-one center in the NBA? _____

- A way to find the number-one center is to rank the players in each category from 1 (first) to 10 (last). The rank for teamwork would be 1—Jabbar (48); 2—Ewing (36); 3—Laimbeer, Parrish (35); 5—Daugherty, Sikma (34); 7—Olajuwon (33); 8—Gilmore (32); 9—Rollins (31); 10—Malone (27). Note that no 4 is listed because two players tie for number 3. Rank the other categories, total the ranks for each player, and order the players.
- Another way to select the best center is to determine who has the highest point total from the five characteristics. Find this total for each player and enter the results under "Total Points." Use those results to rank the players in the next column.
- Some people prefer to *weight* or count some of the categories more than others because they think that teamwork and offense, for example, are more important than the others. To find a weighted score, you might double the numbers for teamwork and offense and add them to the values in the other categories. For example, Daugherty's weighted total would be 41 + 37 + 2(34) + 2(38) + 41 = 263 (see table 2). Complete the column "Weighted Points" using this procedure. Now select two or more categories that you feel are more important, weight them, and complete the column "Your Weighted Points"; then rank the players.

Table 2

Player	Total Points	Ranking	Weighted Points	Ranking	Your Weighted Points	Ranking
Daugherty			263			
Ewing						
Gilmore						
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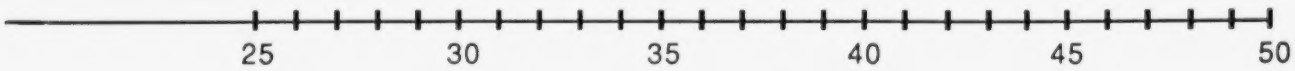
Comparing These Methods

- How do the three methods compare? _____
- Which method did you like the best and why did you like it? _____
- Can you think of another way to rank the players? _____
- Who is your choice for the number-one center in the NBA? _____ Why? _____
- Who is the second best? _____ Why? _____
- If you were Malone's agent, which statistics would you use to stress that he was more valuable than Ewing? _____
  Why? _____

Using Box-and-Whisker Plots

General managers might prefer to pay higher salaries to players who consistently have high ratings. A box-and-whisker plot of the ratings can help show consistency. To make a box plot, arrange the values for an individual player in order of size. Find the median, or middle, number and plot it on a number line. For example, Patrick Ewing's ratings are 47, 45, 39, 36, and 34, with a median of 39. Plot the lower quartile for the numbers, which is halfway between 36 and 34, or 35. Plot the upper quartile, which is 46, the average of 47 and 45. Draw a box from the lower-quartile value to the upper-quartile value (from 35 to 46). Draw a line, or whisker, from the box to the smallest rating, 34, and from the box to the largest value, 47. The line through the middle of the boxes is the middle, or median, rating. Complete the box plots below showing the range and the distribution of the five ratings given in table 1 for each player.



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Olajuwon

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Rollins

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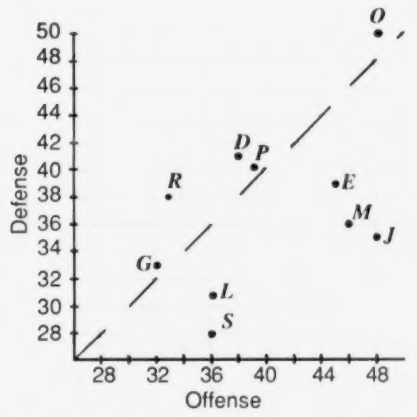
Another way to look at the ratings is to look at combinations of the categories. Suppose you want to see who rates the highest in offense and defense only. A *scatterplot* of the offensive and defensive ratings for each player can help. For example, the point ***P*** corresponds to the ordered pair (39,40), the offensive and defensive ratings, respectively, for Robert Parrish.



Notice that the line $y = x$ has been drawn in on the scatterplot:

9. Where is the point for a player who has an equal offensive and defensive rating? _____

10. The highest possible rating for both offense and defense is represented by the point (50,50), since 50 is the maximum possible rating in any category. The point for the player who is best in both categories should be closest to the line $y = x$ in the upper-right corner. Which center has the highest rating for both offense and defense? _____

11. Which players rate higher in offense than defense? _____

    Where are the points representing those players? _____

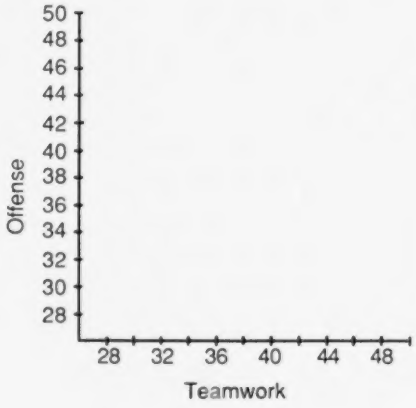
12. What do the players above the line have in common? _____

Complete the scatterplot for teamwork and offense.

13. Are the centers stronger in offense or in teamwork? _____ Why? _____

14. Which player is rated the highest in both offense and teamwork? _____

15. Sometimes statisticians make scatterplots of all possible pairs of categories and call this a matrix of the scatterplots. They look for any patterns they can find and to see which players rate the highest most often. How many different scatterplots would you have to make for a complete matrix of the NBA centers in which all pairs of categories are compared? _____



Using the Distance Formula

The distance formula can be used to find an algebraic solution for the player who has the highest rating in both offense and defense. The distance between two points (x_1,y_1) and (x_2,y_2) =

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

To find the distance from ***P***(39,40) for Robert Parrish to the point (50,50), the highest possible rating, we have

$$\begin{aligned}\sqrt{(50 - 39)^2 + (50 - 40)^2} &= \sqrt{11^2 + 10^2} \\ &= \sqrt{121 + 100} \\ &= \sqrt{221}.\end{aligned}$$

Robert Parrish is $\sqrt{221}$ units from the highest possible rating in both categories. You could simplify or use a calculator to approximate $\sqrt{221}$, but the numbers are easily compared in this form.

Find similar distances for each of the other players. Complete the chart for the comparison of both (offense, defense) and (teamwork, offense). Do the algebraic results using distance agree with your answers using the scatterplots?

_____

Which method do you prefer? _____

Player	(Offense, Defense) Distance	(Teamwork, Offense) Distance
Daugherty		
Ewing		
Gilmore		
Jabbar		
Laimbeer		
Malone		
Olajuwon		
Parrish	$\sqrt{221}$	
Rollins		
Sikma		

Did You Know That . . .

- the formula for the number of possible combinations of n categories taken r at a time is $n!/(r!(n - r)!)$?
- Pat Ewing is currently the highest paid active NBA player, with a salary of $2,750,000 a year?
- the *Sporting News* 1987 Basketball Special rated Olajuwon as the number-one center, followed by Malone, Laimbeer, Ewing, and Sikma, in that order? The results were calculated by using the TENDEX formula: points plus rebounds plus assists plus blocked shots plus steals minus turnovers minus missed field-goal and free-throw attempts, with the total divided by minutes played and that quotient divided by the game pace.
- Statistics or data analysis is required for a college degree in many fields, including sociology, psychology, history, occupational therapy, accounting, economics, industrial engineering, and geography?
- the NBA center with the best 1986 free-throw shooting record is Jack Sikma of the Milwaukee Bucks, who made 87 percent of his free throws?
- the tallest NBA player is Manute Bol, who is 7 feet, 6 inches tall, and the shortest is Tyrone Bogues at 5 feet, 3 inches, both on the Washington Bullets?

Can You . . .

- find the number of different combinations if you compare three, four, or five of the categories from table 1 at a time?
- think of any other situations that could be analyzed in the same way as the NBA centers?
- rate the centers by finding the median for each category, assigning a plus 1 for a score above the median and a minus 1 for a score below the median, and then finding the total for each player?
- set up a similar rating scale for movies?

Answers

Player	Total of Rankings	Order	Total Points	Ranking	Weighted Points	Ranking	(Offense, Defense) Distance	(Teamwork, Offense) Distance
Daugherty	23	4	191	5	263	5	$\sqrt{225}$	$\sqrt{400}$
Ewing	20	2	201	3	282	3	$\sqrt{146}$	$\sqrt{221}$
Gilmore	39	10	172	10	236	10	$\sqrt{613}$	$\sqrt{648}$
Jabbar	21	3	202	2	298	1	$\sqrt{229}$	$\sqrt{8}$
Laimbeer	26	6	189	6	260	6	$\sqrt{557}$	$\sqrt{421}$
Malone	24	5	195	4	268	4	$\sqrt{212}$	$\sqrt{545}$
Olajuwon	17	1	215	1	296	2	$\sqrt{4}$	$\sqrt{293}$
Parrish	28	7	178	7	252	7	$\sqrt{221}$	$\sqrt{346}$
Rollins	37	9	174	8	238	9	$\sqrt{433}$	$\sqrt{650}$
Sikma	32	8	174	8	244	8	$\sqrt{680}$	$\sqrt{452}$

Olajuwon is the number-one center according to two methods of ranking, and Jabbar is highest in the weighted ranking. Another answer could be acceptable also if the r for ranking can be explained.

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3. Daugherty, Ewing, Olajuwon, Jabbar
4. Olajuwon
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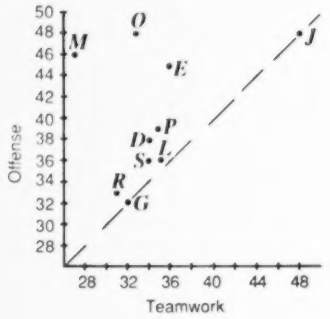
6. Malone
7. Gilmore and Rollins
8. Daugherty
9. On the line $y = x$
10. Olajuwon
11. Laimbeer, Sikma, Ewing, Malone, Jabbar. Below the line $y = x$
12. They rate higher in defense than offense.
13. Offense, all points below the line
14. Jabbar
15. 10



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student math notes

Statistical Decision Making

A panel of five experts rated the centers in the National Basketball Association (NBA) on five characteristics: aggressiveness, shooting range, teamwork, offense, and defense. The players were rated on each characteristic from 10 for excellent to 1 for poor. Table 1 gives the totals in each category for ten of the players.

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Using these results, who would you choose as the number-one center in the NBA? _____

- A way to find the number-one center is to rank the players in each category from 1 (first) to 10 (last). The rank for teamwork would be 1—Jabbar (48); 2—Ewing (36); 3—Laimbeer, Parrish (35); 5—Daugherty, Sikma (34); 7—Olajuwon (33); 8—Gilmore (32); 9—Rollins (31); 10—Malone (27). Note that no 4 is listed because two players tie for number 3. Rank the other categories, total the ranks for each player, and order the players.
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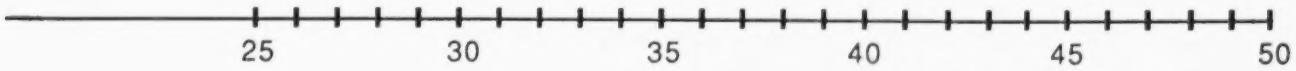

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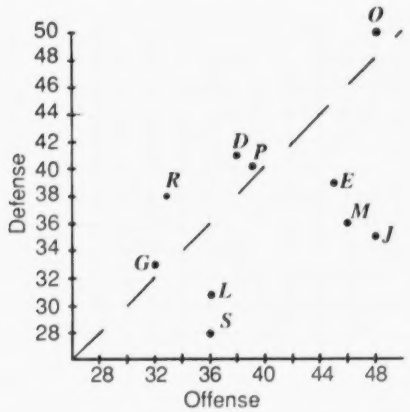

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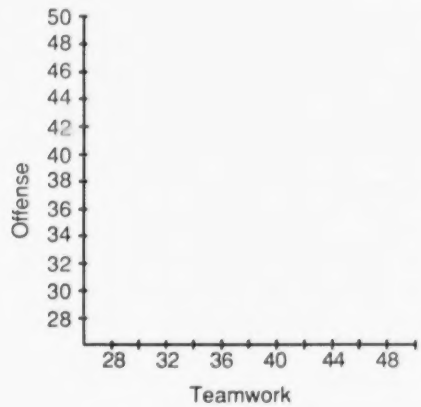

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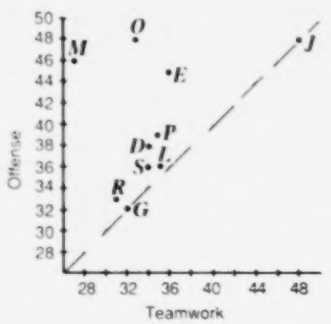

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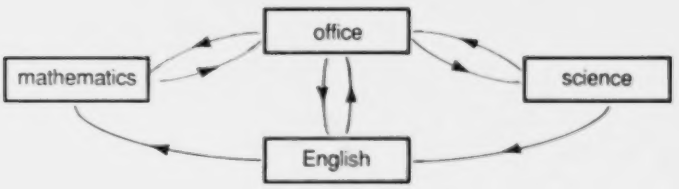
Directed Graphs

At U-Rah High School the intercom lines are connected according to the following graph:



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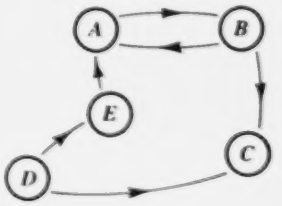

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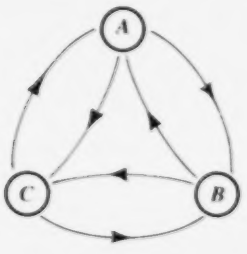

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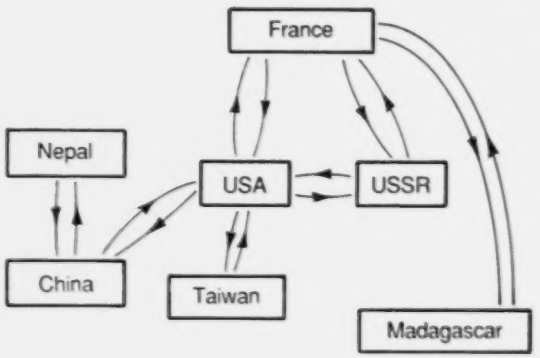

9. How would the USSR communicate with Madagascar? _____
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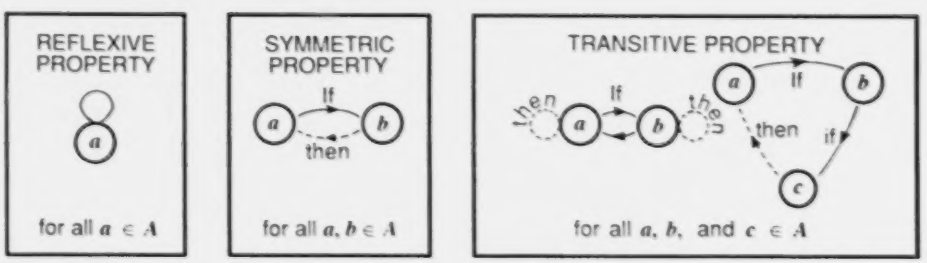

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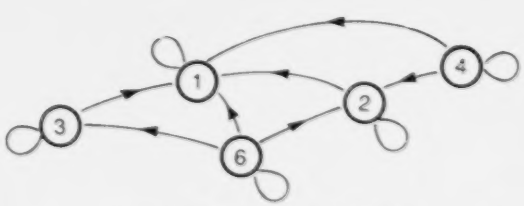

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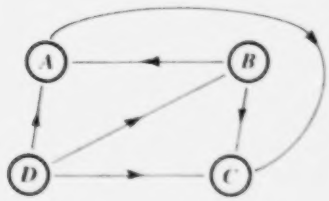

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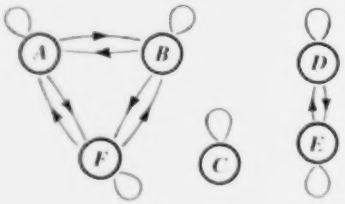

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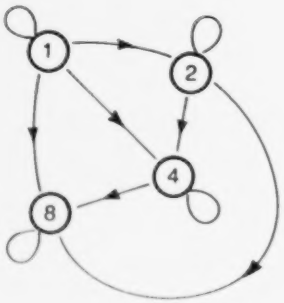

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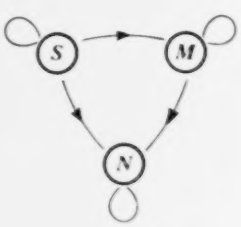

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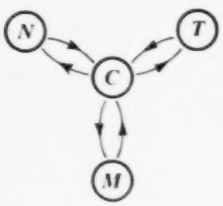

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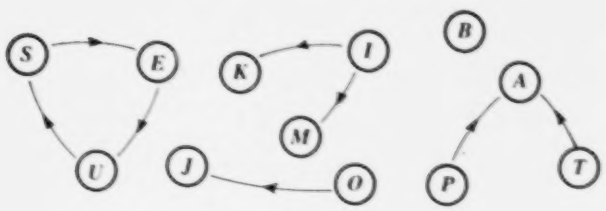

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Can You . . .

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2. No
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4. An arrow connects the English department to the office.
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8. (*a*) Won 1, lost 2; (*b*) Because A has won one game while D has won two, you might expect D to win. Also D defeated E and E defeated A, so that you might expect D to win; (*c*) 6; (*d*) B and D have each won two games. (*e*) C has won no games.
9. One way is through France to Madagascar
10. One way is through USA to China
11. Nepal, Madagascar; USA
12. 42
13. *a*) The transitive property
    b) The reflexive, symmetric, and transitive properties
    c) The reflexive and transitive properties
14. *a*) Reflexive and transitive
    b) Symmetric
15.
16. *a*) a is the grandmother of c.
    b) b is the daughter of a.
    c) c is the child of b.
    d) c is the grandchild of a.
17. *a*) The relation is reflexive, symmetric, and transitive.

b) None

c) None

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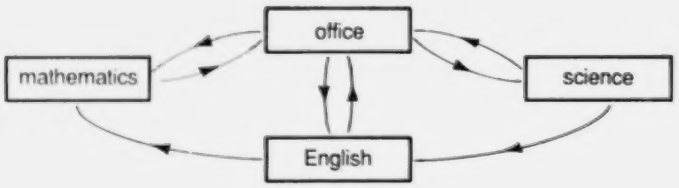

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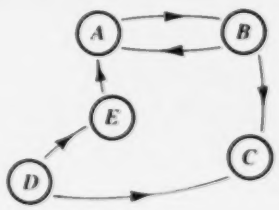

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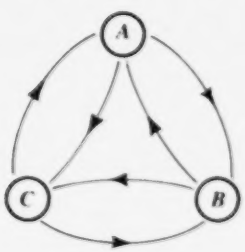

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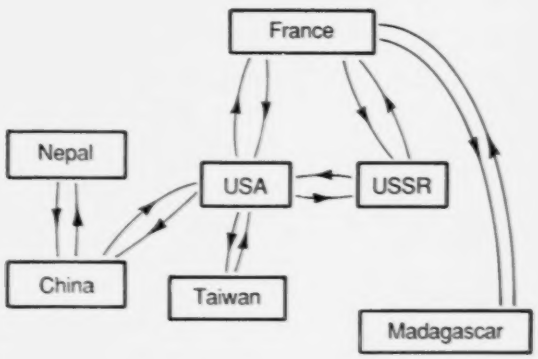

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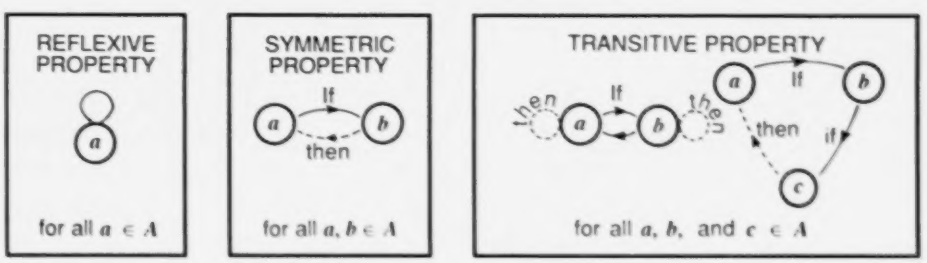

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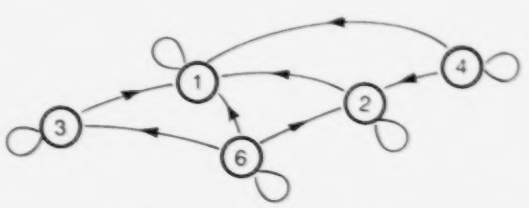

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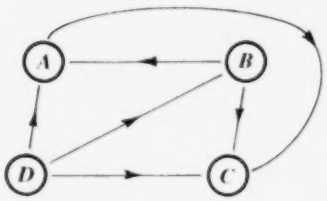

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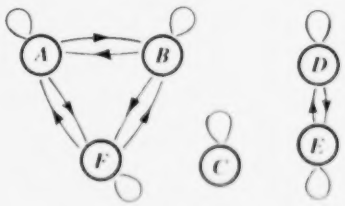

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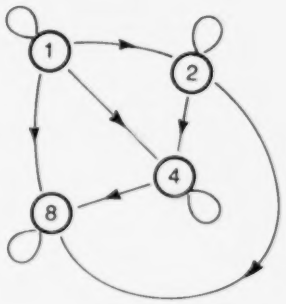

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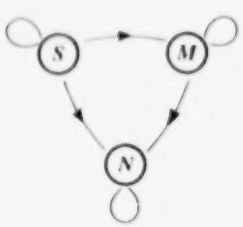

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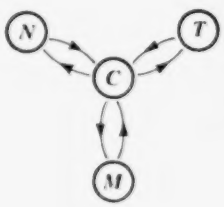

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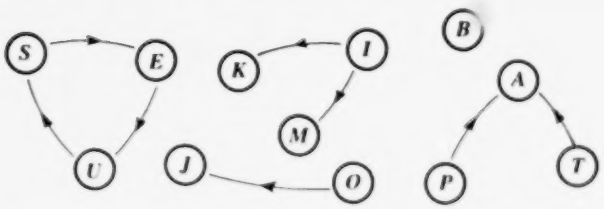

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7.

	A	B	C	D	E
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D	0	0	1	0	1
E	1	0	0	0	0

8. (*a*) Won 1, lost 2; (*b*) Because A has won one game while D has won two, you might expect D to win. Also D defeated E and E defeated A, so that you might expect D to win; (*c*) 6; (*d*) B and D have each won two games. (*e*) C has won no games.
9. One way is through France to Madagascar
10. One way is through USA to China
11. Nepal, Madagascar; USA
12. 42
13. *a*) The transitive property
    b) The reflexive, symmetric, and transitive properties
    c) The reflexive and transitive properties
14. *a*) Reflexive and transitive
    b) Symmetric
15.
16. *a*) *a* is the grandmother of *c*.
    b) *b* is the daughter of *a*.
    c) *c* is the child of *b*.
    d) *c* is the grandchild of *a*.
17. *a*) The relation is reflexive, symmetric, and transitive.

b) None

c) None

18. (*a*) None; (*b*) all

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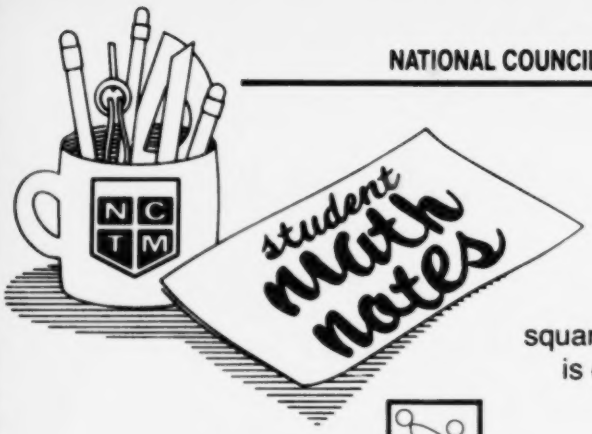

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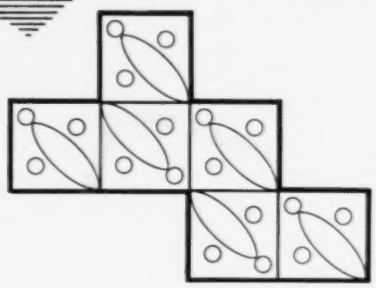

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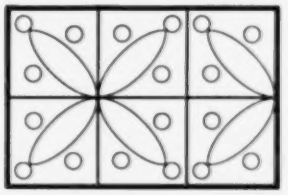

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- Find the total cost of the tile and framing for each mosaic shown above.

Design 1 = _____ Design 2 = _____

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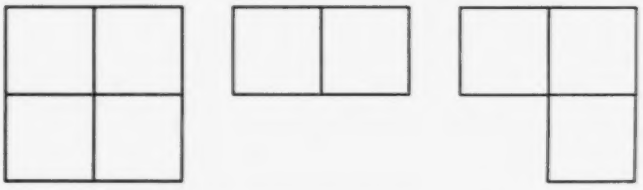

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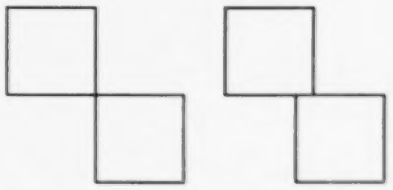

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Investigating Perimeter

You will need grid paper and fifty to one hundred square tiles or squares cut from construction paper. Let the length of the side of a square be one unit. Note that the area of each arrangement is equal to the number of squares. Work with a group of classmates to investigate the perimeter of certain arrangements found by putting squares together.

The editors wish to thank Judy Mumme, mathematics department of the University of California at Santa Barbara, for writing this issue of *NCTM Student Math Notes*.

Investigating Perimeter—Continued

- Verify the results in table 1 for one, two, and three squares.
- Explore different arrangements of squares to determine the perimeter for each.
- Explore the range of perimeters possible for each number of squares to find the arrangements that produce the maximum and minimum perimeters.
- Use grid paper to sketch the arrangements for the maximum and minimum perimeters.
- Record the maximum and minimum perimeters for the area arrangements in table 1.
- Look for patterns in the table and on the grid paper to see if you can predict the maximum and minimum perimeter of an arrangement. You should extend table 1.

Table 1

Area (No. of Sq.)	Maximum Perimeter	Minimum Perimeter
1	4	4
2	6	6
3	8	8
4	10	
5		10
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		

1. Can you find an arrangement for which the perimeter is an odd number? If so, draw the arrangement. If not, explain. _____

2. Predict the *maximum* perimeter of arrangements of the following numbers of squares:

   40 _____ 75 _____ 100 _____

3. Make a conjecture for finding the *maximum* perimeter of arrangements of any number of squares.

4. Now, try to predict the *minimum* perimeter of arrangements of the following numbers of squares:

   36 _____ 48 _____ 55 _____

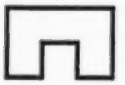
5. Make a conjecture for finding the *minimum* perimeter of arrangements of any number of squares.

Investigating Areas

Using the same rule for arranging squares, explore the minimum and maximum *area* of an arrangement of squares with a *given perimeter*.

- The arrangement shown in the grid has a perimeter of 10. Outline five other arrangements that have a perimeter of 10. Indicate the area of each arrangement on the grid.

- The arrangement shown on this grid has a perimeter of 12. Outline as many other arrangements as you can that have a perimeter of 12. Indicate the area of each.



- Continue to explore different arrangements of squares that produce a given perimeter and determine the area of each arrangement in order to find arrangements that produce the *minimum* and *maximum* areas. Record these values in table 2.

Table 2

Perimeter	Minimum Area	Maximum Area
4	1	1
6	2	2
8	3	4
10	4	
12		9
14		
16		
18		
20		
22		
24		
26		

6. Predict the *minimum* area for the following perimeters:

   30 _____ 36 _____ 44 _____

7. Make a conjecture for finding the *minimum* area.

   _____

   _____

   _____

8. Predict the *maximum* area for the following perimeters:

   30 _____ 36 _____ 42 _____

9. Make a conjecture for finding the *maximum* area.

   _____

   _____

   _____

Did You Know That . . .

- using the same rule as before, only one way exists to arrange one square, one way to arrange two squares, two ways to arrange three squares, and five ways to arrange four squares?

- you can add squares to an arrangement to increase the area and decrease the perimeter?
- this investigation can be extended to three dimensions using cubes to explore the relationship between surface area and volume?
- design and packaging engineers also have to consider the relationship between perimeter and area in maximizing profit while minimizing cost. Similar concerns face civil engineers and housing developers when they lay out streets and lot designs.

Can You . . .

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- find various arrangements of squares whereby adding a square will decrease the perimeter?
- find a rule for maximum and minimum perimeters for a given number of equilateral triangles?
- find a shape that will produce the maximum area for a given perimeter?
- find the arrangements of five squares that will fold into an open-top box?
- find an arrangement of cubes whereby adding a cube to an arrangement increases the volume and decreases the surface area?

Answers

Page 1

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Page 2

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2. 82, 152, 202
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4. 24, 28, 30
5. Answers will vary with sophistication: Let a be the number of squares; let g be the greatest integer in $\sqrt{a}$ (the whole-number square root). If $\sqrt{a} = g$, then $p = 4\sqrt{a}$. If $a \leq g^2 + g$, then $p = 4g + 2$. If $a > g^2 + g$, then $p = 4(g + 1)$.

Page 3

Table 2

Min: 1, 2, 3, 4, 5, 6, . . .

Max: 1, 2, 4, 6, 9, 12, 16, 20, 25, . . .

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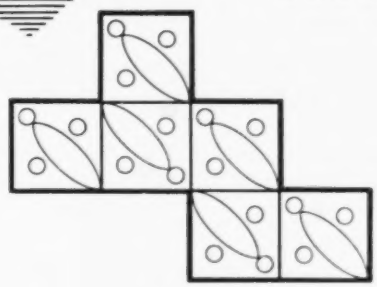

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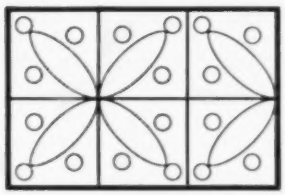

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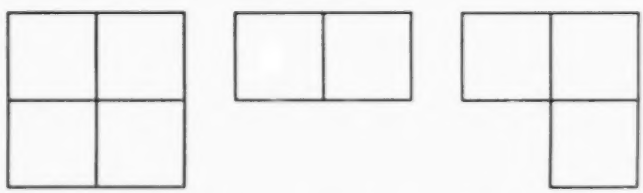

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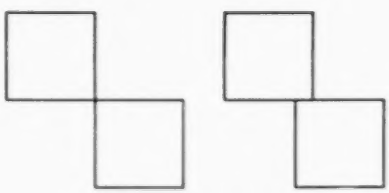

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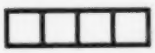

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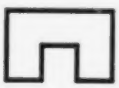

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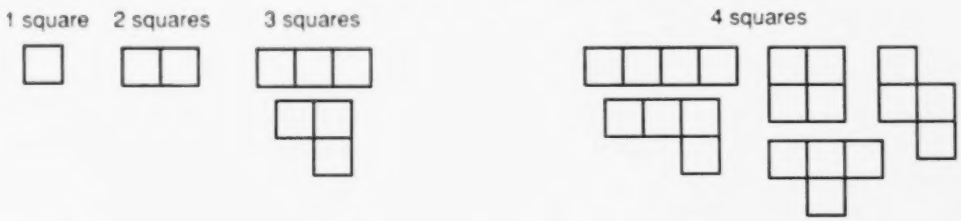

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